A RE-EXAMINATION OF THE BEGINNING OF MOVEMENT FOR COARSE GRANULAR BED MATERIALS

by: C.R. Neill

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FOREWORD AND ACKNOWLEDGEMENTS

The investigation described was conducted at the Hydraulics Research Station, Wallingford, England during the winter of 1967-68, while the author was on leave from the Research Council of Alberta, Edmonton, Canada. The assistance of the Director and staff of the Hydraulics Research Station and the use of their experimental facilities are gratefully acknowledged. The stimulus and encouragement provided by the papers and lectures of Dr. M.S. Yalin (formerly of the Hydraulics Research Station) must also be mentioned.

This report was prepared by Mr. C. R. Neill during his temporary attachment to the Hydraulics Research Station. The views expressed and the conclusions drawn are those of the author; they do not necessarily reflect the official views of the Station.

ABSTRACT

Certain questions on the Shields entrainment function that require clarification are posed and discussed. It is shown that if the 'beginning of movement' is considered in numerical terms, the logical result is a 3-parameter relationship expressing a dimensionless grain displacement rate as a function of the original Shields parameters.

Flume experiments on low rates of movement of uniformized coal grains, glass spheres and coal mixtures are described and analyzed. It is shown that reasonably consistent results can be obtained if a numerical criterion for beginning of movement is adopted, and that for uniform grains little significant difference in Shields' entrainment parameter can be detected for grain Reynolds' numbers exceeding 70 or so. Empirically, the beginning of movement for uni-modal mixtures of moderate dispersion can be estimated by inserting the D_{50} size by weight into the expression for uniform materials.

Suggestions are made for further research on low transport rates of coarse materials that might improve insight into bed-material transport functions. The need for a more fundamental approach to transport of mixtures is recognized. A similarity approach to a transport function, slightly different from previous approaches of the same kind, is outlined briefly.

The analysis and presentation of grain-size distributions for mixtures is discussed in an appendix.

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1. THEORETICAL DISCUSSION

1.1. Questions on the Shields function

In 1936, A. Shields, an American engineer studying in Germany, published[1] his criterion for beginning of movement of uniform granular material on a flat bed. His 'entrainment function', now widely accepted as one of the fundamentals of sediment transport, seems to have been introduced to English readers by H. Rouse. It may be written:

$$\left(\frac{\tau_o}{\gamma_s' D}\right)_{crit} = f\left(\frac{V_* D}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{or} \quad Y_{crit} = f\,(X) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1a)$$

where τ_o = bed shear stress

V_* = shear velocity = $\sqrt{\tau_{o/\rho}}$

γ_s' = submerged specific weight of grains = $g(\rho_s - \rho)$

D = characteristic diameter of grains

ν = kinematic viscosity of fluid

and Y_{crit} denotes 'critical' value of Y for beginning of movement.

The reasoning behind the function may be found in several textbooks, and will not be repeated. The combination Y may be interpreted as a measure of the ratio of moments of lift plus drag to grain weight. The combination X may be interpreted as a measure of the ratio of grain diameter to the thickness of the laminar sub-layer (real or hypothetical) on the bed.

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[1] References are listed on p. 28.

Shields reasoned that at sufficiently high values of X (practically, with grains in the gravel category), fluid viscosity should have negligible influence and Y_{crit} would tend to a constant value.

Fig. 1 shows the data points originally plotted by Shields and the curve drawn by Rouse, constituting the 'Shields diagram' as usually quoted. (Actually, Shields drew not a single curve, but a broad band, for reasons to be explained later.) The limiting value of Y_{crit} for high values of X is usually quoted as 0.06 or thereabouts.

Since 1936, many data have been added by other experimenters, and the theoretical and experimental aspects of initial movement have been reviewed by several writers[2,3,4]. Egiazarov[5] presented a Shields-type diagram including 220 points, showing a wide scatter. This accumulation of data has done little to clarify a number of questions concerning the Shields diagram. Among these are the following:

1. Exactly what did Shields mean by "beginning of movement"? Is the condition amenable to objective definition?

2. Above what value of X is Y_{crit} effectively constant?

3. What are the effects of grain shape and orientation? What is the effective diameter of an irregular grain?

4. What is the effect of mixtures of different grain sizes? Can the effective size of a mixture be defined?

1.2. Definition of beginning of movement

Shields said "the beginning of movement is experimentally to be determined by extrapolating the bed-load transport curve to the time bed-load movement ceases". It is not clear from his paper whether he actually followed this procedure throughout. In any case, he used some points by earlier experimenters, who had determined initial movement by visual assessment only.

Recent writers[6] have noted that Shields' limiting value of Y_{crit} for the fully-rough turbulent zone (0.06) corresponds to a low but measurable rate of transport. At values of 0.03 and even less, occasional movement of single grains may be obtained. In fact, the more carefully an experimental search for the real 'beginning of movement' is conducted, the more elusive does the concept appear. We shall therefore consider in some detail certain arguments for modifying or abandoning it.

M. S. Yalin (in lectures at the University of Alberta, 1966-67) pointed out that in order to observe similarity in initial movement experiments on a series of beds, each composed of grains of the same shape but of different single sizes, the following conditions should be specified:

- area of observation $A \propto D^2$;
- time of observation $t \propto D/V_*$;
- observe same number of grains displaced in each case.

Carrying this argument a stage further, we can write

$$n = f\,(V_*,\ D,\ \gamma_s',\ \rho,\ \nu) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

where n = number of grains displaced per unit area of bed per unit time,

and V_* = time-average value of shear velocity.

Arranging non-dimensionally, we obtain instead of the Shields function

$$\frac{nD^3}{V_*} = f\left(\frac{\rho V_*^2}{\gamma_s' D}, \frac{V_* D}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\text{or} \quad N = f\,(Y, X) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3a)$$

Eq. (3), derived by considering similarity in experimental determinations of the beginning of movement, allows for the experimental difficulty that we cannot in practice observe the 'beginning' of movement, only some degree of established movement. In further informal discussion with the writer on this point, Yalin has suggested that 'beginning of movement' might be defined (by general convention) as corresponding to a certain numerical value of the non-dimensional combination N, just as boundary layer thickness is conventionally defined in a somewhat arbitrary way.

Now consider the physical circumstances associated with the 'beginning of movement' in turbulent flow. (The argument that follows requires some expansion for the case of very small grains, where there is a recognizable laminar sub-layer at the beginning of movement. For the sake of brevity, this will be omitted.) Velocities and shear stresses at the bed, and therefore lift and drag forces on the individual grains, are subject to random fluctuations with respect to time (Fig. 2). At very low rates of grain movement, only occasional peak values are competent to displace grains. But if we accept that mean values of drag and lift, as averaged over short time periods, follow a statistical distribution function (Fig. 3), it is apparent that rare grain movements may occur even with very low values of the mean shear. Therefore the curve of n versus τ_o must be very flat for low values of n (Fig. 4), and it must be practically impossible to decide visually where to draw the line and call it 'beginning of movement'.

A few investigations on the distribution of turbulent fluctuations may be mentioned. Pantelopulos[7,8] has presented both laboratory and field data on the fluctuation of streamwise velocities, averaged over short time periods, near a rough bed. These values tended to scatter symmetrically about their overall average, according to a Gaussian frequency distribution. Einstein and El Samni[9] measured lift fluctuations on a fixed bed of inverted hemispheres, and found an approximately Gaussian frequency distribution. Popovich and Hummel[10] measured shear fluctuations in smooth-turbulent flow with a laminar sub-layer, and also found an approximately Gaussian distribution. There seems to be a certain discrepancy between Pantelopulos' and Einstein's results, in that one might expect the statistical distribution of shear and lift fluctuations in fully rough flow to be skewed, if the velocity fluctuations are symmetrically distributed. However, the experimental accuracy may be insufficient to permit detection of small degrees of skew.

Concerning the magnitude of turbulent fluctuations near a rough bed, Pantelopulos obtained, for short-period average velocities at a point, a standard deviation of the order of 1/6 of the long-term mean value. If we take +4 standard deviations as close to the limit of practical interest (values in excess of this occurring only 0.003% of the time), then the maximum value should be approximately 1.67 times the long-term mean, and the maximum value of shear stress should be nearly three times the long-term mean. Somewhat similar figures have been inferred by several writers from comparing experimental shear values at 'beginning of movement' with theoretical figures based on the assumption of a non-fluctuating drag.

1.3. Lower limit of 'self-simulating zone'

The zone in which Y_{crit} is independent of X is sometimes called the 'self-simulating zone', for this reason: if we model a rough-bed channel geometrically (without distortion), scale the velocity according to the Froude law (velocity scale squared = length scale), and scale the bed-material geometrically (without change of density), then initial movement of bed-material will occur at similar stages in model and prototype, provided the model bed material is large enough to lie within the self-simulating zone.

According to the Shields diagram (Fig. 1), the lower limit of the self-simulating zone occurs at X = 1000 approximately, corresponding to D = 10 mm for a specific gravity of 2.65. But it is usually accepted that the fully-rough turbulent zone begins at $\frac{V_* k}{\nu} \approx 70$, and that for a flat uni-granular bed, $k \approx D$. Therefore, should Y not be independent of X for X > 70 approximately, corresponding to D = 2 mm for a specific gravity of 2.65? It will be shown subsequently that such, in fact, may well be the case. Meyer-Peter and Muller[11] previously reported effectively constant Y values for X > 50 or so. Iwagaki[12] obtained a constant value, from theoretical reasoning, for X > 150. Goncharov[13] and Egiazarov[14] assume somewhat similar values.

1.4. Effects of grain shape and orientation

Shields concluded that 'rounded' and 'sharp-edged' grains gave higher Y_{crit} values than 'angular' grains. It seems questionable, in view of various possible factors making for scatter in his results, whether a significant difference due to grain-shape was actually established.

The tendency of natural sands and gravel particles to adopt preferred orientations under certain conditions of transport (imbrication) is well-known. The effect of this on the Shields function does not seem to have been studied systematically, nor was it investigated in the writer's experiments.

1.5. Effect of mixtures

Although the 'Shields curve' is usually applied to grains of more or less uniform diameter, in fact Shields' original diagram included a few points for quite widely-graded mixtures. This was the main reason why he drew a band rather than a single curve through his points, as previously mentioned. In plotting these points for mixtures he used the median (D_{50}) size by weight to calculate X and Y. This he justified on the basis of earlier researches in Germany by Casey and by Kramer[15]. He did not consider mixtures in any detail, nor did he question Casey's and Kramer's unsatisfactory methods of characterizing mixture gradings.

Kramer's experiments[15] were made with three different mixtures in the 0.1 to 5 mm range. He defined critical conditions for a mixture as that point at which 'general movement' began: i.e. the largest grains began to move. He related critical values of Y (based on D_{50}) to a so-called 'uniformity modulus' of the mixture, but did not consider X. The investigation was somewhat inconclusive and his paper was criticized extensively. In adapting Kramer's data, Shields apparently did not use the 'general movement' criterion, but rather what Kramer called 'weak movement', where only the smallest grains were mobile.

Pantelopulos[7,8] discussed transport of mixtures in terms of turbulence and probability theory, and reported experiments on coal and sand-gravel mixtures (size range 0.1 to 13 mm). His principal conclusions regarding beginning of movement may be summarized as follows:

(i) the middle-size fractions in a mixture move at Y values (calculated from the properties of the fraction in question) comparable to those required for a uniform bed;

(ii) the smaller sizes are sheltered, and require higher Y values; and the larger sizes are exposed, and move at lower Y values (calculating Y from the properties of the fraction considered);

(iii) for the mixtures tested, effective threshold or initial movement conditions for the mixture as a whole occurred at approximately the same shear stresses that would have produced initial movement of a uniform bed of $D = D_{50}$ (median diameter by weight of mixture);

(iv) under certain conditions the transport behavior of a widely graded mixture can be adequately represented (in models) by a more narrowly-graded mixture or even by a uniform material.

The writer's data, quoted subsequently herein, support the first three of the above conclusions. The last should probably be regarded skeptically.

Egiazarov[14,16], adopting 0.06 as the Y_{crit} value for uniform material in the self-simulating zone ($X > 200$), gave a formula for initial movement of any fraction in a mixture:

$$\frac{\tau_c}{\gamma_s' D_i} = \frac{0.10}{\left(\log 19 \frac{D_i}{\overline{D}_{avg}}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where D_i = mean diameter of fraction,

and $\overline{D}_{avg}$ = an 'average' diameter for the mixture, defined in a rather complex way.

The right-hand side reduces to 0.06 approximately when $D_i = \overline{D}_{avg}$. The formula was derived by considering the velocity distribution very close to the bed, assuming that the effective roughness of the bed was determined by $\overline{D}_{avg}$. Egiazarov presented a curve showing good agreement between the formula and experimental data by Pantelopulos and others. The following objections, however, may be raised:

(i) it does not seem reasonable to apply the theoretical velocity distribution at points situated below the summits of the largest grains. In this region the physical picture must be confused, and the action of the flow on smaller grains must depend on geometrical details of their arrangement;

(ii) Egiazarov's method of determining $\overline{D}_{avg}$ depends on knowledge of the sizes in motion and cannot readily be determined from a grading curve for the original bed mixture.

Egiazarov considers that initial movement of the mixture 'as a whole' is given by substituting D_{50} for D_i in Eq. (4). Since D_{50} is usually less than $\overline{D}_{avg}$ as he defines it, he obtains somewhat higher values of Y_{crit} for mixtures than for uniform material. He points out that a mixture may be entirely in the self-simulating zone even if it contains very small grains, because its roughness is determined by the larger grains.

Gessler[17] defined the critical shear stress for a given grain as that value of the time-average shear stress for which the "probability of being eroded equals the probability of remaining at rest". This definition seems incomplete: we may, for example, ask "Over what period of time and over what area of bed?" Using data from experiments on the degradation and 'armoring' of mixed beds, he produced a modified Shields diagram with curves labelled for various such probabilities, in which the traditional Shields curve has a 'probability' of approximately 0.5. If this means that at any point of the bed the instantaneous shear exceeds the critical value for 50% of the time (which may be a misinterpretation of Gessler's meaning), it would seem to require a rate of grain movement very much larger than Shields meant by 'beginning of movement'.

Rakoczi[18] experimented on initial movement of sand-gravel mixtures containing luminescent tracer fractions. His conclusions were more or less in line with the first two of Pantelopulos, as summarized previously herein.

Burgess and Hicks[19] experimented on the stability of stone mixtures on slopes under wave attack. They concluded that different gradings of mixture had equal resistance to attack if their D_{50} sizes by weight were equal: provided the mixture was selected for its D_{50} size, its sieve curve was of little importance.

Ramette and Heuzel[20] studied initial movement of gravel in a river, using radioactive tracer pebbles. The bed contained sizes from sand to 200 mm, with D_{50} = 20 mm, and the labelled pebbles were in three fractions of approximately 24, 40, and 60 mm mean diameter respectively. The smallest labelled fraction was observed to move at a Y value of approximately 0.035, and the larger fractions at lesser values (all calculated from the properties of the fraction considered). They concluded that "in the middle of a range of pebbles having a sufficiently extended grain-size curve, the entrainment coefficient (Y_{crit}) varies in inverse proportion to diameter and tends towards a limit in the neighbourhood of 0.020 for the largest pebbles". This is more or less in line with the findings of the experimental studies previously cited, but the last point is not in accord with Egiazarov's formula (4).

The results of the writer's experiments on widely-graded coal mixtures, which will be quoted subsequently, agree substantially with the findings of Pantelopulos, Rakoczi, Burgess and Hicks, and Ramette and Heuzel.

1.6. Summary of questions

It is evident that the four questions on the Shields function discussed in 1.2 to 1.5 above are inter-related. The effects of grain shape and of mixtures, and the range of the self-simulating zone, cannot be properly determined without establishing objective definitions of the beginning of movement. The existing mass of subjective observations can be of little help, because it remains uncertain whether variations in Y_{crit}, ascribed to variations in X, grain shape, or grading, were in fact due to variations in N.

1.7. Comparison of 'beginning of movement' in air and in water

Values of Y_{crit} quoted for the beginning of bed movement in water are usually in the range of 0.03 to 0.06 for the fully rough zone, and seldom as low as 0.02 even for the supposed trough of the Shields curve at $X = 10$. It seems at first sight curious that values usually quoted for sand grains in air, on the basis of experiments by Bagnold[21] and others, are in the range of 0.01 to 0.02. The following may constitute a partial explanation:

Consider again the proposed numerical criterion of movement:

$$N = f\,(Y, X) \qquad \text{(3a)}$$

where $N = \dfrac{nD^3}{V_*}$ and n = no. of grains displaced per unit area per unit time.

Now, for the same grains and for equal values of Y, V_* will be larger in air than in water by a factor of the order of 30. Therefore, for equal values of N, n must also be larger by the same factor. Suppose a certain observed condition is defined as 'beginning of movement' in water. If the corresponding value of N is measured and reproduced in air, the resulting condition of movement will appear much higher to the casual observer, because of the shortened time-scale. Therefore, subjective visual assessment of 'beginning of movement' will tend to result in lower N values and therefore lower reported Y_{crit} values in air.

2. EXPERIMENTS AND RESULTS

2.1. Previous experiments at Edmonton (1966)

In 1966 the writer conducted flume experiments at the University of Alberta on the beginning of movement of uniform gravel beds, with the aim of developing design criteria in terms of mean velocity rather than shear stress. These were written up in internal report of the Research Council of Alberta dated January, 1967. A summary of the results was presented in a paper to the 1967 IAHR Congress[22]. This paper was criticized by the general reporters at the Congress. The writer then re-examined the data and concluded that he had previously mis-interpreted his results to some extent, partly as a result of assessing 'beginning of movement' subjectively. A partial correction was later published in the Journal of Hydraulic Research[23].

2.2. Experiments at Wallingford (1967-68)

The original aim of the experiments conducted at Wallingford was to continue the program begun at Edmonton, by examining initial movement of coarse widely-graded mixtures. In the event a considerable time was spent on further experiments with uniform-size materials, as it became evident that problems of defining beginning of movement (see preceding theoretical discussion) could not be side-stepped. The theoretical concepts previously discussed however, only became clear as the experiments proceeded, so that in retrospect the experimental planning was not ideal.

The experiments were conducted in a tilting flume 80 ft. x 3 ft. x 1 ft., with steady uniform flow. Depth of flow was generally about 0.5 ft. The following series of tests on low rates of movement were carried out:

Series A: Tests on 4 near-uniform sizes of crushed coal, ranging from approximately 20 to 3 mm diameter. The object was to determine shear stresses for low rates of movement, and in particular for two conditions designated 'lower critical' and 'upper critical' stages respectively. The largest size of coal was tested at three different depths; the others at a single constant depth.

Series B: Tests on 4 widely-graded coal mixtures, to determine shear stresses for low rates of movement of the several fractions in each mixture and for each mixture as a whole.

Series C: Tests on two sizes of uniform glass balls, for comparison with series A.

Table 1 summarizes the material properties and the ranges of flow conditions for these tests. A few additional preliminary tests were run to check shear distributions and flow measurements in the experimental flume.

2.3. Experimental methods - Wallingford experiments

Arrangement of beds. A 50 ft. length of the flume was laid with 38 mm - 19 mm gravel, tamped flat. A small area of each mobile test material was inserted in the center of the flume width near the downstream end, flush with the surrounding gravel (Fig. 6). The areas used for the various materials, and the transitions areas around them (to avoid sudden changes in roughness) are shown in Fig. 5. The mobile areas were proportioned to the dominant size of the material, as outlined in section 1.2. Fig. 7 shows the grain-shape characteristics of the series A materials on a Zingg diagram.

Grading curves for the four coal mixtures are shown in Fig. 8. They may be described as follows:

M.1. Approximately log-normal grading by weight, range of sieve sizes approximately 50 to 5 mm.

M.2. Approximately log-equal grading (equal fractions by weight in each sieve interval), range 50 to 5 mm.

M.3. Mixture 1 modified by addition of 20% by weight in 5 to 1 mm range.

M.4. Approximately log-normal grading, range 50 to 1 mm.

The mixtures were made up from $\sqrt{2}$ interval fractions. Fractions larger than 5 mm diameter were each spray-painted a different color. The components were mixed as uniformly as possible and the mixtures spread out flat (but not smooth) so that there was no appreciable projection of the larger sizes.

Fig. 9 compares, for mixture M.1, the grading curve by weight with a measured surface distribution by area (before erosion). The percentage of area occupied by the smaller fractions was very small, partly due to unavoidable settling through the surface layer.

Determination of bed shear stress. Slope was measured by manometers, and velocity profiles by miniature current-meter (10 mm propeller). Generally profiles were measured on the flume center-line at each end of the mobile area, and sometimes at other points. They were plotted on semi-log paper, and the origin of y adjusted to produce best straight-line fit near the bed (usually about D/4 below the tops of the grains). Local time-average shear stress was then calculated from the profiles according to

$$V_* = \frac{V_2 - V_1}{5.75 \log y_2/y_1} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

and $$\tau_o = \rho V_*^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

and compared with the value calculated from

$$\tau_o = \gamma h S \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

valid for the central part of a wide channel. (Although the width/depth ratio was only 6, the bed was much rougher than the walls, so that a wall correction was judged an unnecessary refinement.)

Where the dominant grain-size of the mobile area was not greatly different from that of the surrounding gravel, reasonably good agreement was obtained between the two determinations. With the smaller uniform-size materials, however, the velocity profiles generally gave somewhat lower values, as might be expected from boundary layer considerations. The question arises of whether the profiles were taken close enough to the bed (minimum y was approximately 7 mm) to give a correct estimate of the shear in a changing boundary layer. In some cases a break showed up in the profile at a certain height, indicative of a change in the boundary layer associated with the change in roughness, but in other cases it did not. The theoretical problems involved here were not deeply investigated, and it was assumed that the effective shear on the bed could be estimated with sufficient accuracy by considering the two values determined by flume slope and velocity profiles respectively. In test A.8, the smallest uniform size of coal was tested with a fixed approach bed of its own grain-size roughness, but no significant difference from test A.7, where no such approach was provided, was apparent. Velocity values at a point were always determined from the mean of ten 10-second counts.

Determination of conditions of grain movement. In general, conditions of movement at a given flow were defined by counting the number of grains of each size displaced downstream of the mobile area in specified time intervals. (It would have been more logical to count all displacements whether internal or external, but considerably more difficult.) The time interval was adjusted to the size of the grains, as outlined in section 1.2. Attention was concentrated particularly on two conditions, designated herein as lower and upper critical stages. For the uniform-size materials, lower critical stage was the first stage at which a count could conveniently be made, and corresponds to an initial N value (Eq. 3a, section 1.2) of the order of 3×10^{-6}. (It is necessary to say the initial value, because at low displacement rates of randomly oriented natural grains, the initial rate is not maintained after the most unstable grains have been removed.) Upper critical stage, corresponding to an N value of the order of 15 x 10^{-6}, was more or less the first stage at which a roughly steady displacement rate was maintained for a considerable time. It produced fairly rapid degradation of the mobile areas.

For the mixtures, it proved difficult (for a variety of practical reasons) to characterize these critical stages numerically in a satisfactory way, but an attempt was made to produce correspondence to the criteria for uniform materials. The adopted stages may be described as follows:

Lower - grains of several of the middle fractions displaced at rates comparable to a uniform material of equivalent size; no significant movement of largest and smallest grains.

Upper - more or less steady displacement of all but the largest grains; occasional displacement of largest grains under long-continued flow; fairly rapid degradation of mobile area.

Test procedure. Flow was increased in small steps until a few grain displacements could be counted in an interval of a few minutes. At each step, slope was adjusted for uniform flow. In successive steps, counts were made over periods varying from several minutes to several hours. Where a long count resulted in significant bed degradation, the bed was sometimes re-made before continuing at higher flows. Velocity profiles were not taken at every step, but generally at lower and upper critical stages.

2.4. Summary and analysis of results

Table 2 summarizes the experimental results at lower and upper critical stages for each test. Appendix 5.3 contains samples of the actual observations over a range of flows in each test. These results are now analyzed briefly in terms of the questions posed in section 1.1.

Definition of beginning of movement. The experiments showed that while in principle it is possible to adopt a consistent numerical definition according to kinematic similarity as outlined in section 1.2, in practice the labor involved in observation would be great. The difficulties are caused mainly by random turbulent fluctuations - some of a long period - and the random nature of grain shapes and environments. Therefore the criteria adopted for 'lower and upper critical stages' were only roughly consistent. For instance, values of N calculated from initial displacement counts of the several uniform coal fractions, at lower critical stage, varied from 1.7 to 6.0 x 10^{-6}. To give an idea of the physical significance of the mean figure, for 10 mm pebbles it represents a displacement rate of approximately 10 grains per minute per square meter of bed. The associated transport rate is extremely small, because the average hop length of the grains is also very small. The adopted upper critical stage is of the order of 5 times greater in terms of displacements, but relatively higher in terms of transport, because the hop lengths are also greater.

Table 3 summarizes values of X and Y for the uniform coal fractions at both stages.

To check whether depth of flow had any influence on measured shear stresses, coal fraction T was tested at 3 different depths, with results as follows (extracted from Tables 1 and 2):

Depth of flow	Y for lower critical stage of movement
61 mm	0.030
145 mm	0.028
170 mm	0.034

It is concluded that the effect of depth is not significant.

Lower limit of X for effectively constant Y_{crit}. Table 3 indicates that in the range of coal sizes tested as uniform beds, the supposed dependence of Y_{crit} on X was not convincingly demonstrated; also that values for the lower critical stage were considerably below the Shields curve. It appears that for practical modelling purposes the self-simulating zone may be taken as extending throughout the gravel size range (i.e. as low as 2 mm), and possibly lower.

It seems possible that the gradually increasing values for Y_{crit} plotted by Shields over the range X = 50 to X = 500 or so may have reflected an inconsistent criterion of movement. If his criterion was in effect less severe for the larger grain sizes - which tends to be the result of visual observations in a single channel - then the plot would tend to show a false trend of Y_{crit} increasing with X (i.e. with D).

Spherical vs. irregular grains. 2 sizes of uniform glass balls, arranged loosely with a more or less random projection, were tested in a similar manner to the coal fractions. Results are given in Table 4.

These results suggest that at the lower critical stage, there is little significant difference between spheres and irregular grains if equivalent spherical diameter (volume basis) is adopted as the characteristic size of the latter. At the upper critical stage there seems to be a significant difference. It was observed that the test areas of spheres appeared to disintegrate quite rapidly after a relatively small number of grains had been displaced.

Widely-graded mixtures. The behaviour of the 4 coal mixtures under increasing shear was similar, and roughly as follows.

There was no noticeable movement until grains of the middle fraction, or a fraction close to the middle, started moving. At a slightly higher shear all sizes up to about half the maximum were liable to displacement. At about twice the shear stress required for the 'lower stage' of movement (middle sizes just mobile), all sizes were mobile and the bed degraded rapidly. Intermediate shear stresses tended to produce a paved or armored bed, where the remaining smaller grains were sheltered from further erosion by the immobile larger grains.

Table 5 shows best estimates of Y_{crit} (in terms of D_{50} by weight) for the 4 mixtures, at lower and upper critical stages as described in section 2.3. The figures compare reasonably well with those for uniform materials. It is uncertain whether the discrepancies for mixture M.4 should be regarded as significant: it is possible that somewhat more severe criteria were applied to this mixture, owing to difficulty in tracking movement of the smaller grains.

The main difficulties in adopting any quantitative definition of beginning of movement for the mixtures arose from (i) the impossibility of counting the smallest grains of mixtures M.3 and M.4, and (ii) the fact that the rate of displacement at the higher stages was largely determined by the previous history of flow increases, which varied considerably between individual tests.

A few determinations were made of the values of Y, in terms of $\gamma_s' D$ for the fraction in question, at which some of the largest fractions in the mixtures were first displaced. The lowest value obtained was 0.019, corresponding closely to the lower limit of 0.020 found by Ramette and Heuzel[20] in river investigations (see section 1.5).

3. CONCLUSIONS AND RECOMMENDATIONS

3.1. Conclusions

The following conclusions are suggested by this somewhat limited investigation:

1. An absolute threshold of grain movement cannot be observed in turbulent flow. The threshold can only be defined in numerical terms, as an average number of grains eroded per unit area per unit time. The Shields 2 - parameter function is thereby replaced by a 3 - parameter one of the form:

$$N = f\,(X,\ Y) \qquad \text{(3a)}$$

where N = dimensionless erosion rate $\frac{nD^3}{V_*}$;

n = number of grains eroded per unit area per unit time;

and X and Y are Shields' parameters.

2. For a given low value of N, Y is not significantly affected by X if $X > 70$ approximately. In practical terms, this means that for quartz grains the self-simulating zone extends down to a grain diameter of the order of 2 mm.

3. For design of a stable bed of more or less uniform stones, a suitable value of Y is 0.03. This corresponds to an N value of the order of $3/10^6$, giving negligibly small rates of displacement for most practical purposes. The commonly quoted Shields value of 0.06 gives an N value in the order of 5 to 10 times greater, corresponding to rates of displacement that cannot be regarded as negligibly small.

4. Eq. 3a above may partly explain certain apparent discrepancies between published Y_{crit} values for 'beginning of movement' in air and in water respectively.

5. For N values of the order of 3×10^{-6}, there is no significant difference in Y values between uniform spheres and irregular grains of the same average equivalent spherical diameter. For higher values of N, the uniform spheres appear to be more easily eroded.

6. For uni-modal mixtures characterized by a $\sqrt{D_{84}/D_{16}}$ value of up to at least 2.5 or so, the beginning of movement of the mixture as a whole may be regarded - at least empirically - as governed by the D_{50} size by

weight. A value of Y of 0.03 (in terms of D_{50}) will result in displacement of the middle sizes at very low rates, comparable with those of a similar uniform size under the same shear.

For mixtures with a $\sqrt{D_{84}/D_{16}}$ value of the order of 2, significant displacement of all sizes will not occur until the dimensionless shear parameter Y_{50} reaches 2 to 3 times the above 'lower critical' value. Intermediate shears will tend to produce an 'armored' bed. For more extended mixtures, the factor may be higher, but this has not been proved.

7. For scour protection, a graded mixture of rip-rap stone would appear to be approximately equivalent to a single-size material of $D = D_{50}$. The shape of the grading curve appears to be of little importance with reference to direct erosion of the surface layer. (Erosion or percolation of the underlying material is a separate question.)

3.2. Suggestions for further research on low transport rates of coarse materials

(i) Uni-granular beds. Determine the curve of N vs. Y by counting the number of grain displacements for a succession of shears, each applied for a long enough time to give good average values. The curve may have the form shown in Fig. 4.

Test at least 2 different sizes and 2 different densities of grain (minimum of 3 materials).

Determine whether there is any significant difference between results on a flat and on a wavy or rippled bed.

Measure average hop lengths of grains between displacement and coming to rest, and plot dimensionless hop length l/D vs. Y. Determine whether it depends on grain/fluid density ratio as well as on Y. Compare flat and rippled beds.

If possible, repeat some of these experiments in a wind tunnel and determine whether similar N values are obtained for equal values of Y.

(ii) Mixtures. A fundamental experimental approach to low transport of mixtures raises great difficulties. The following questions may indicate some possible lines of enquiry:

1. How is the distribution by size of displaced grains related to their original areal distribution on the surface?

2. Under what conditions is transport selective, resulting in an 'armored' bed?

3. Can an N - Y function be defined for different fractions?

4. How is hop length affected by the presence of a mixed rather than uniform bed?

5. How do segregation of sizes, imbrication (preferred orientation) of grains, and surface roughness projection affect behaviour?

6. Is there any good hydraulic reason for log-normal or other 'natural' grading?

It would seem advisable to investigate the problem further at a theoretical level. Some points requiring clarification are:

1. How are shear-stress fluctuations distributed statistically with respect to time at a point?

2. How does the distribution of average shear-stress values vary with the time-period and area considered?

3. What determines the effective roughness height of a mixed bed?

4. How is the local time-average shear-stress modified by a local change in roughness?

3.3. A similarity approach to a bed-material transport function

The following argument, which is marginal to the main purpose of the present report, is given in somewhat abbreviated form.

Consider separate uni-granular beds of diameter D_1, D_2 etc. Kinematic similarity in transport is given by displacement of the same number of grains from geometrically similar areas A_1, A_2 etc. of each bed in kinematically similar times t_1, t_2 etc., and their transport in geometrically similar 'hops' of length l_1, l_2 etc. For similarity

$$A \propto D^2$$

$t \propto D/V$, where V is a characteristic velocity of the flow

and $l \propto D$.

Now weight transport rate (either submerged or dry weights may be used) is proportional to grain specific weight x average grain volume x number displaced per unit area per unit time x average hop length. In symbols

$$g_s \propto \gamma_s \cdot D^3 \cdot \frac{\text{no.}}{At} \cdot l$$

Divide both sides by $\gamma_s D V$

Then $$\frac{g_s}{\gamma_s DV} \propto \frac{D^2}{A} \times \text{no.} \times \frac{D}{Vt} \times \frac{l}{D}$$

which for a specified 'number displaced' satisfies the above similarity requirements

$$A \propto D^2$$
$$t \propto D/V$$
$$l \propto D$$

Thus it is shown that $\frac{g_s}{\gamma_s DV}$ is a natural 'dimensionless weight transport rate' arising from considerations of kinematic similarity, independent of dimensional analysis. The characteristic flow velocity V should be taken at a height y above the bed that is proportional to D. For uni-granular beds and fully rough flow

$$\frac{V}{V_*} = f\ (y/k) = f\ (y/D)$$

so that V_* can be taken as representing a characteristic velocity. This gives $\frac{g_s}{\gamma_s DV_*}$, which is Yalin's 'P'[24]. Einstein's ϕ[25] can be shown to be equivalent to $P \cdot Y^{1/2}$, where Y is Shields' parameter.

Next consider on what parameters P should depend. It consists essentially of two components, a dimensionless displacement rate and a dimensionless hop length. The first should depend on X and Y only, whether we consider the 'beginning of movement' or a higher degree of established movement. In the general case, however, the second component should depend also on $\rho_{s/\rho}$ (Yalin's 'W'[26]), because accelerations and decelerations are involved in the hop process. It is curious that data correlations for transport in water do not seem to show any effect of W. It may be that its effect is negligible for moderate differences in its value, but it ought nevertheless to show up in comparisons between transport in air and transport in water.

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5. APPENDICES

5.1. Notation

Symbol	Definition	Units
A	area of bed under observation or in movement	L^2
D	grain diameter	L
D_i	grain size than which i % by weight is finer (in a mixture)	
D_{50}	median grain-size by weight	
$\overline{D}_{avg}$	'average' grain-size of mixture in Egiarazov's formula	
g	gravitational acceleration	L/T^2
g_s	(dry) weight transport rate per unit width	M/T^3
h	depth of flow	L
k	equivalent sand-grain roughness (Nikuradse)	L
l	average hop length of grains	L
n	number of grains displaced per unit area of bed per unit time	no/L^2T
N	dimensionless form of n : $\frac{nD^3}{V_*}$	-
S	channel slope	-
t	time of observation or movement	T
V	characteristic flow velocity	L/T
V_*	'shear velocity' $= \sqrt{\tau_o/\rho}$	L/T
W	ρ_s/ρ (Yalin)	-
X	grain Reynold's number $= \frac{V_* D}{\nu}$	-
y	height above effective bed surface	L
Y	Shields' parameter $= \frac{\tau_o}{\gamma_s' D} = \frac{\rho V_*^2}{\gamma_s' D}$	-

Y_{crit}	value of Y for 'beginning of movement'	-
γ	unit weight of fluid	M/L^2T^2
γ_s	unit (dry) weight of bed-material	"
γ_s'	submerged unit weight of bed-material = $\gamma_s - \gamma$	"
ν	kinematic viscosity of fluid	L^2/T
ρ	mass density of fluid	M/L^3
ρ_s	mass (dry) density of bed-material	"
τ_o	shear stress (time-average) on bed	M/LT^2
τ_c	bed shear stress at beginning of movement	"

5.2. Analysis and presentation of grain-size distributions for mixtures

In sediment transport literature the treatment of grain-size distributions has often been somewhat illogical and at odds with practice in soil mechanics and geology. This note attempts to clarify some of the considerations involved in plotting distributions and describing them numerically. It should be read in conjunction with Table 6 and Figs. 12 and 13, showing a hypothetical distribution analyzed in various ways.

Scales of grain size. A logarithmic grain-size scale is universal in soil mechanics and geology, but an arithmetic scale, or parameters based on it, are frequently encountered in hydraulics literature. In nature it is size ratios and not size differences that seem to be significant with respect to mechanical behaviour. To believe otherwise is to suppose that the range from 0.01 to 1 mm has no more significance than the range from 99.01 to 100 mm. From a practical viewpoint, a log scale produces more symmetrical frequency distributions, and enables the fine end of a wide distribution to be shown clearly.

Geologists transform diameter (in mm) into so-called phi units, forming a log scale to the base 2. One mm is taken as zero; 1/2, 1/4, 1/8 mm etc. as 1, 2, 3ϕ, etc; and 2, 4, 8, mm etc. as -1, -2, -3ϕ etc. This scale has the following properties:

(i) phi differences represent diameter ratios;

(ii) arithmetic mean phi corresponds to geometric mean diameter;

(iii) standard deviation of phi corresponds to geometric standard deviation of diameter.

Mean values. Consider the histogram of a bulk sample sieved by weight (Fig. 12). The mean is defined as the first moment of this diagram about the y-axis. If x is the diameter D, we obtain the arithmetic mean size by weight, which can be written $\frac{\Sigma p_i D_i}{100}$ where p_i = % in fraction of mean diameter D_i. This is the measure used for mixtures by several prominent writers on sediment transport (e.g., Meyer-Peter and Muller, Egiazarov, Bagnold).

If we use a log scale for diameter (Fig. 12b and Table 6), then the central tendency should be characterized by the geometric mean diameter, corresponding to arithmetic mean phi. Some writers draw log distribution curves, but quote the above-mentioned arithmetic mean diameter as their characteristic measure. This seems fundamentally illogical, although in many cases there is not a large difference between the arithmetic and geometric means.

Dispersion measures. The term 'sorting coefficient', commonly used by geologists, will be avoided here, because it embodies the illogical feature that 'poor' sorting, i.e. a wide dispersion of sizes, corresponds to a high value of the coefficient.

The standard deviation of a Gaussian or normal distribution can be determined by reading two values from the cumulative distribution curve. Thus

$$\text{standard deviation of phi} = 1/2\ (\phi_{84} - \phi_{16})$$

$$\text{or geometric standard deviation of D} = \sqrt{D_{84}/D_{16}}$$

where D_{84} = '84% finer than' size, etc. (See fig. 13 and Table 6).

For uni-modal distributions that are reasonably symmetrical on a scale of log-D, $\sqrt{D_{84}/D_{16}}$ can be adopted as a practical measure of dispersion, but it should not be called the geometric standard deviation unless the distribution is truly log-normal. It might be called 'dispersion coefficient'.

The 'uniformity modulus' used by Shields and other early writers on sediment transport is highly illogical as a measure of dispersion.

Median and modal values. For a normal distribution of a given quantity, mean value = median (50% smaller than) value = modal (most frequent) value. For uni-modal distributions that are not far from normal, the following approximation holds (Fig. 14)

$$\text{mean} - \text{median} = 1/2\ (\text{median} - \text{mode})$$

If the quantity in question is phi or log-D, mean phi, corresponding to geometric mean D, may be estimated in this way without calculating moments.

If the distribution of a given sample is plotted to both arithmetic and logarithmic D scales, the median size read from each diagram will be almost the same, but the modal sizes may be greatly different (Figs. 12 and 13).

Many sand-gravel mixtures are bi-modal when plotted to a log-D scale (Fig. 15). To quote the D_{mean}, D_{50} and $\sqrt{D_{84}/D_{16}}$ values for such a mixture does not describe it satisfactorily. Therefore some writers are inclined to prefer the modal value or values above other measures of central tendency.

<u>Probability-paper plots</u>. Some engineers like to plot distributions on log-probability paper, on which the '% finer than' scale is distorted so that log-normal curves plot as straight lines. The disadvantages of this type of presentation are that the tails of the distributions assume undue prominence and that the 0 and 100% limits cannot be shown (Fig. 13c).

<u>Surface sampling of river gravels</u>. Sampling of surface size distributions is sometimes adopted as a less troublesome substitute for volumetric sampling. There are two basic methods:

(i) single stones are picked at even intervals along a line or at the intersection points of a square grid, and counted into size classes (Wolman method);

(ii) all stones within a small specified area are picked and counted into size classes.

It may be shown that in the first method the probability of picking a stone in a given size class tends to be proportional to the fraction of surface area occupied by that class. Therefore if '% by number finer than' is plotted against D or phi, the plot gives approximately the cumulative areal distribution of the various sizes. It is not logical to treat the sample as a bulk sample and analyze it by weight.

The second method, plotted similarly, gives the cumulative number distribution of the various sizes on the surface. This does not seem a particularly useful presentation.

Neither method can give a true representation of the bulk distribution of the material available for transport, despite assumptions to this effect in various writings.

Reference. For a fuller discussion of grain-size distributions see

R.L. Folk, 1965. "Petrology of sedimentary rocks". Lecture notes, Univ. of Texas, published by Hemphill's, Austin, Texas.

5.3. Samples of experimental data

Table 7 shows the grain counts obtained in a typical test of a uniform coal bed on a rising sequence of flow intensities. Fig. 16 shows velocity profiles measured in this test at one particular flow.

Table 8 and Fig. 17 show similar data for a typical test of a coal mixture.

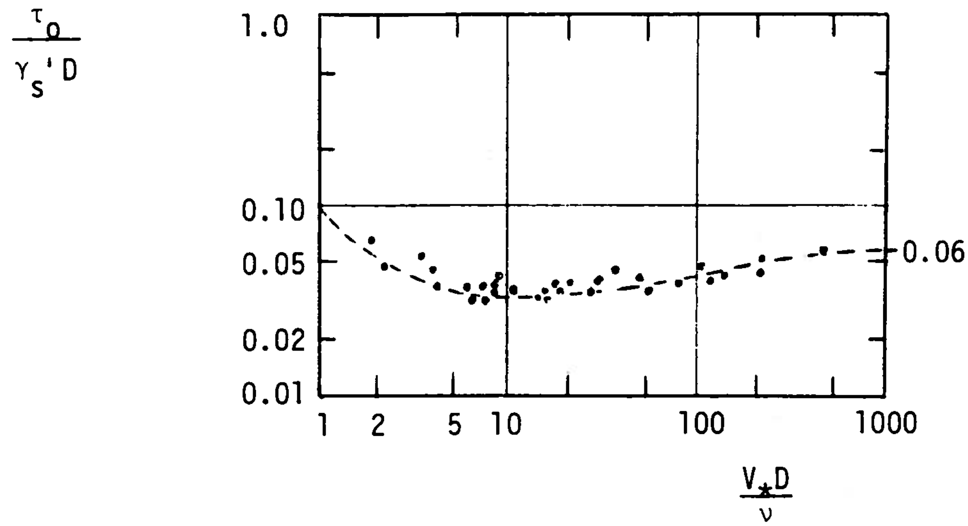


Fig. 1 - Shields diagram.

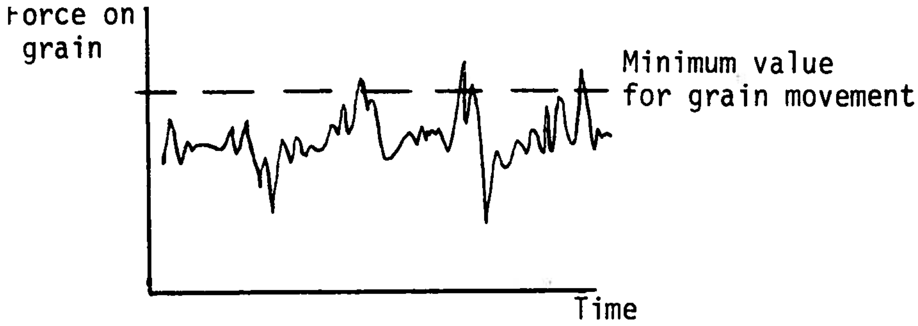


Fig. 2 - Nature of turbulent fluctuations of lift and drag forces on grains at 'beginning of movement'.

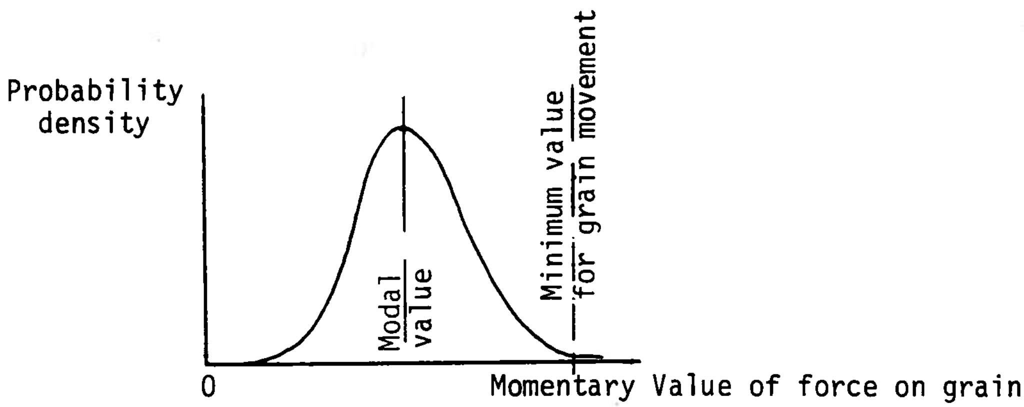


Fig. 3 - Nature of statistical distribution of forces on grains at 'beginning of movement', for a given mean shear.

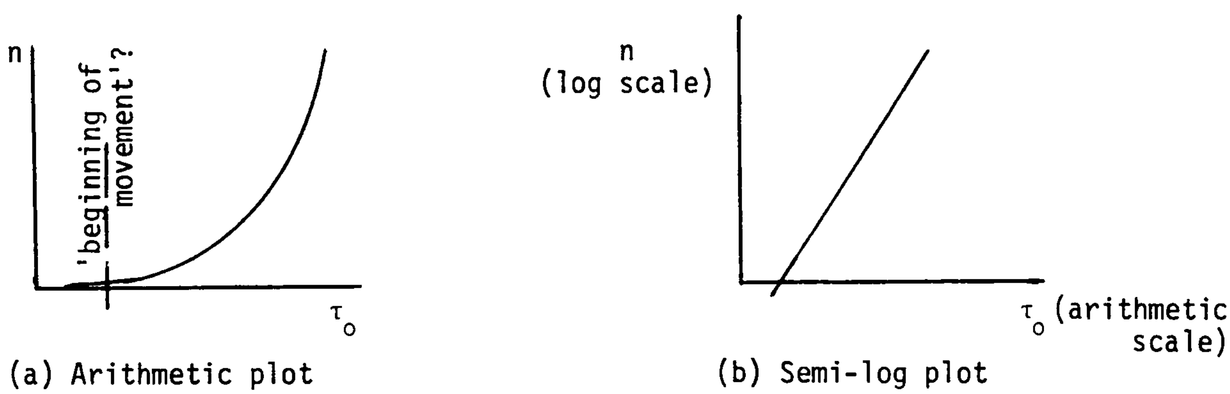
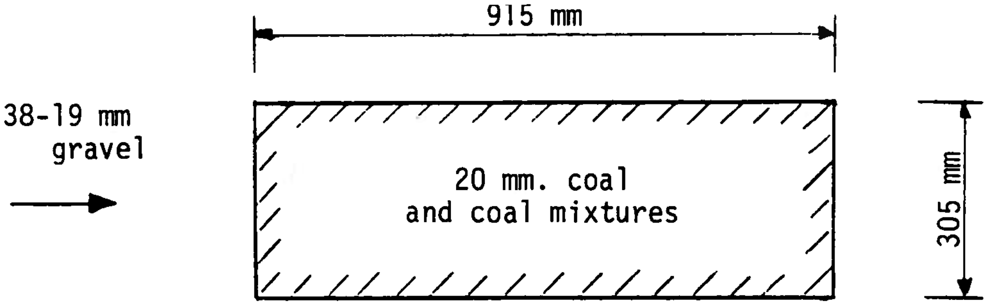
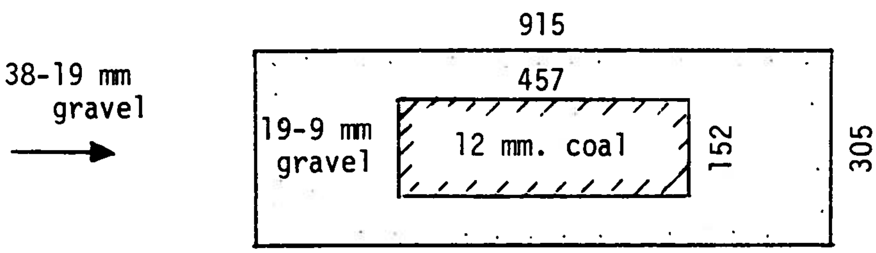


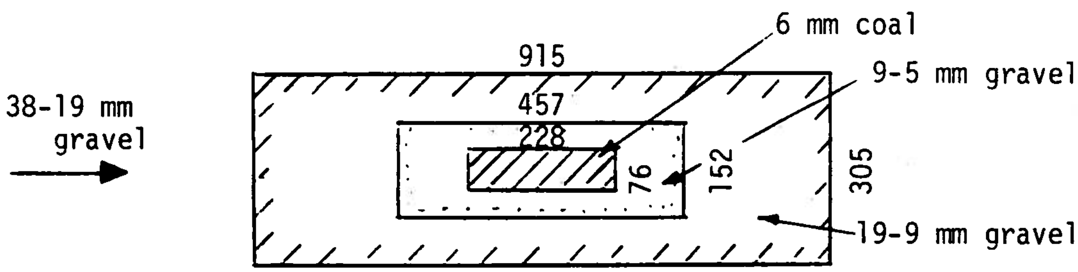
Fig. 4 - Hypothetical curves of rate of grain displacement (n) vs. mean bed shear stress (τ_o).



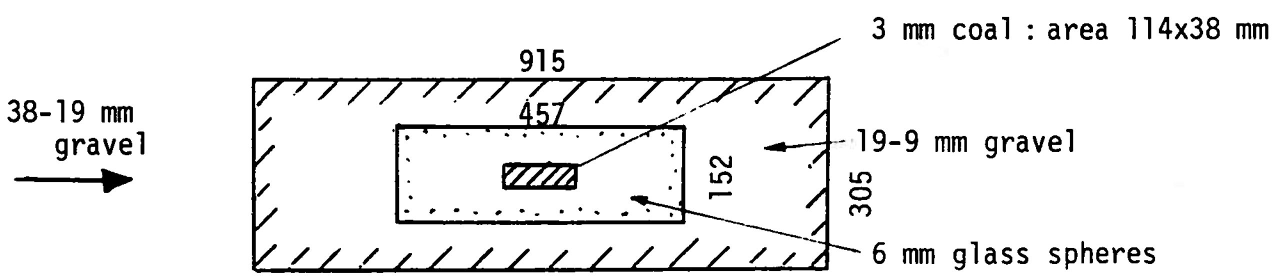
Tests A.1 to A.4 and B.1 to B.4



Test A.5



Test A.6



Test A.7

Fig. 5 - Plan areas of mobile materials and surrounding transitional roughness areas (continued on next page)

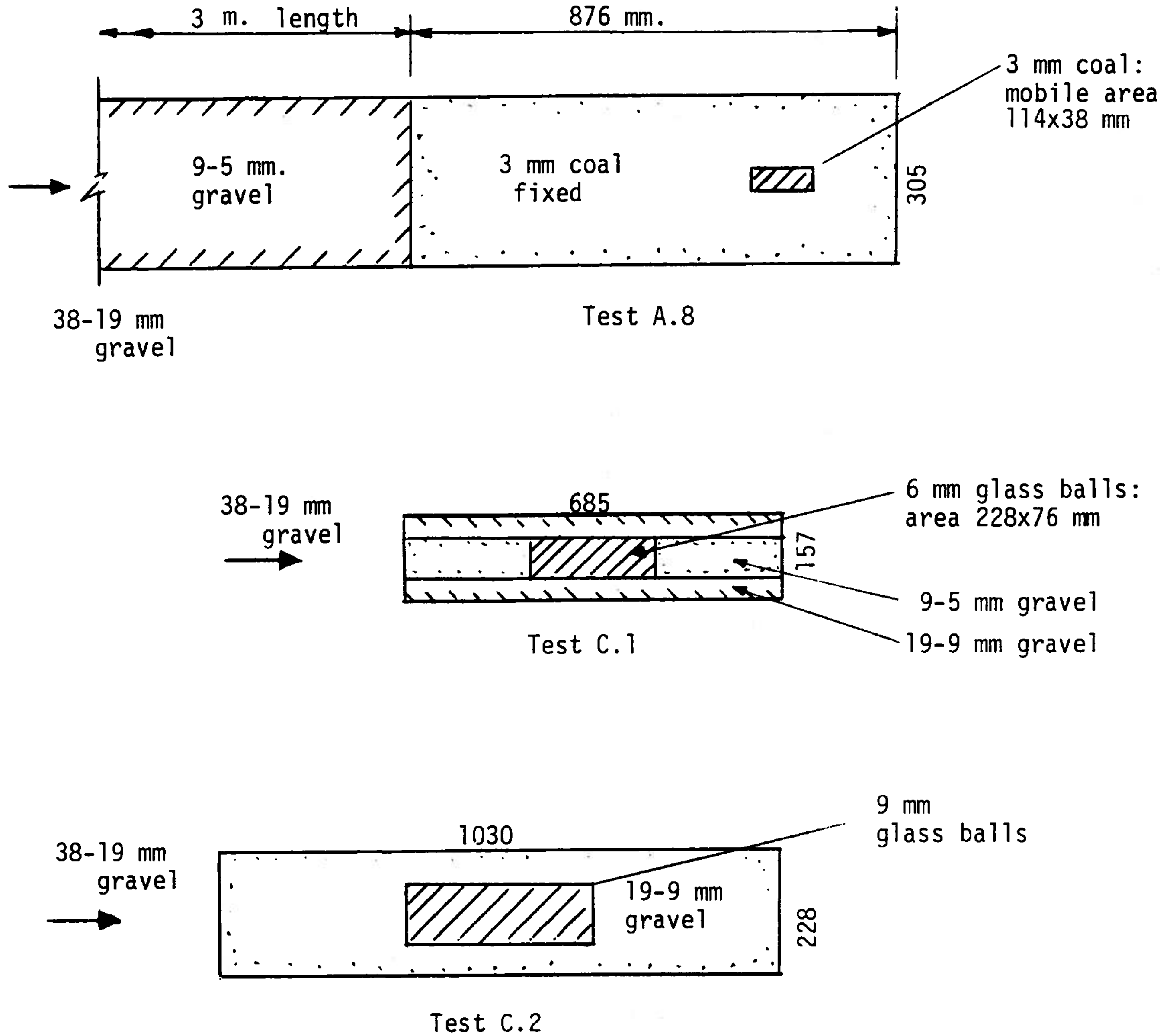


Fig. 5 - Plan areas of mobile materials and surrounding transitional roughness areas (continued from preceding page)

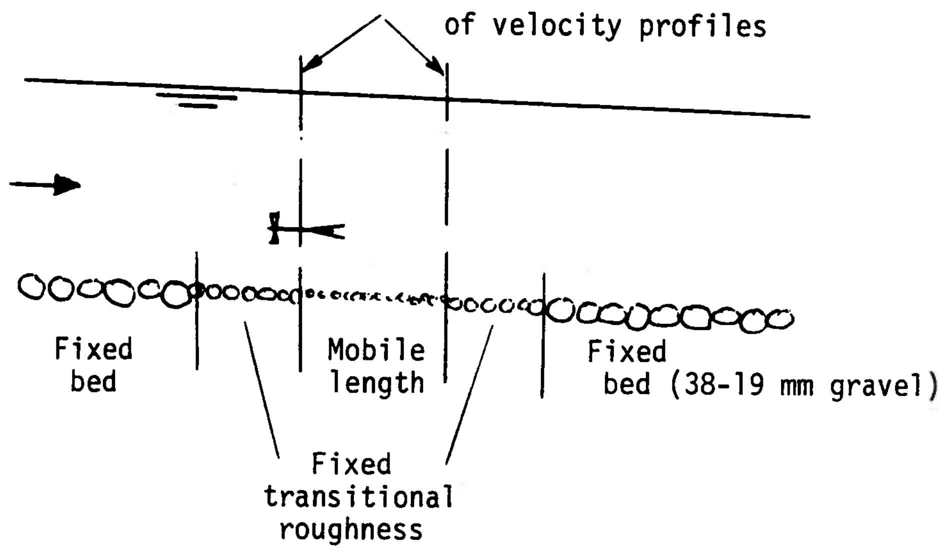


Fig. 6 - Schematic longitudinal section on flume centre-line, showing arrangement of test beds.

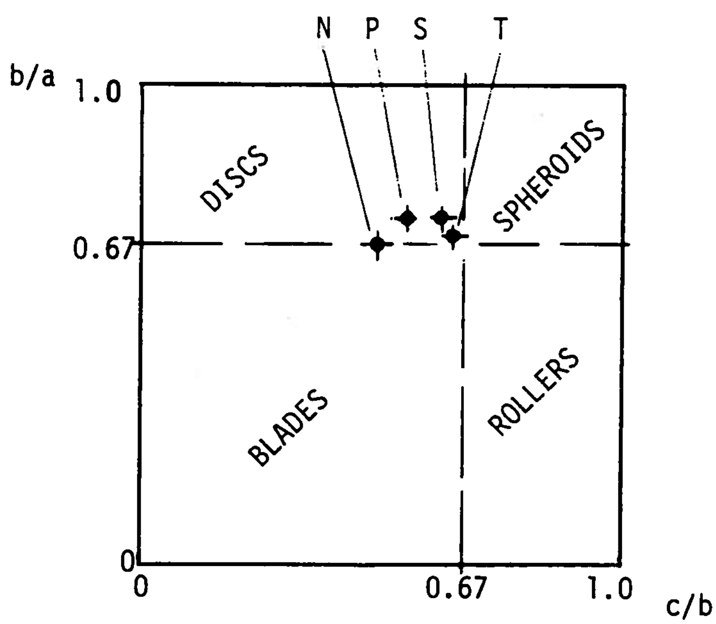


Fig. 7 - Zingg diagram showing grain shape classification of four coal fractions tested as uniform beds.

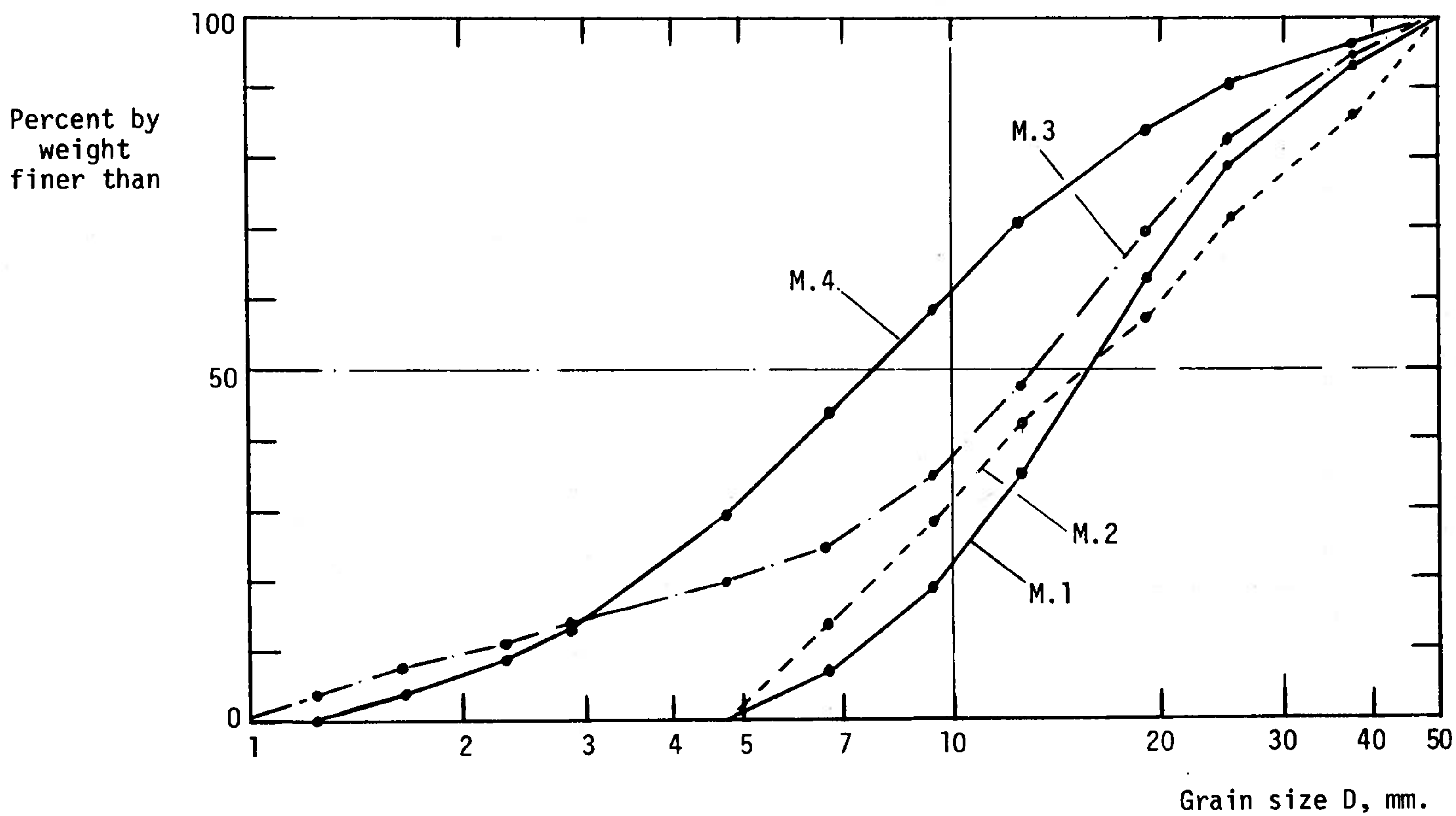


Fig. 8 - Cumulative grain-size distribution curves by weight of four coal mixtures.

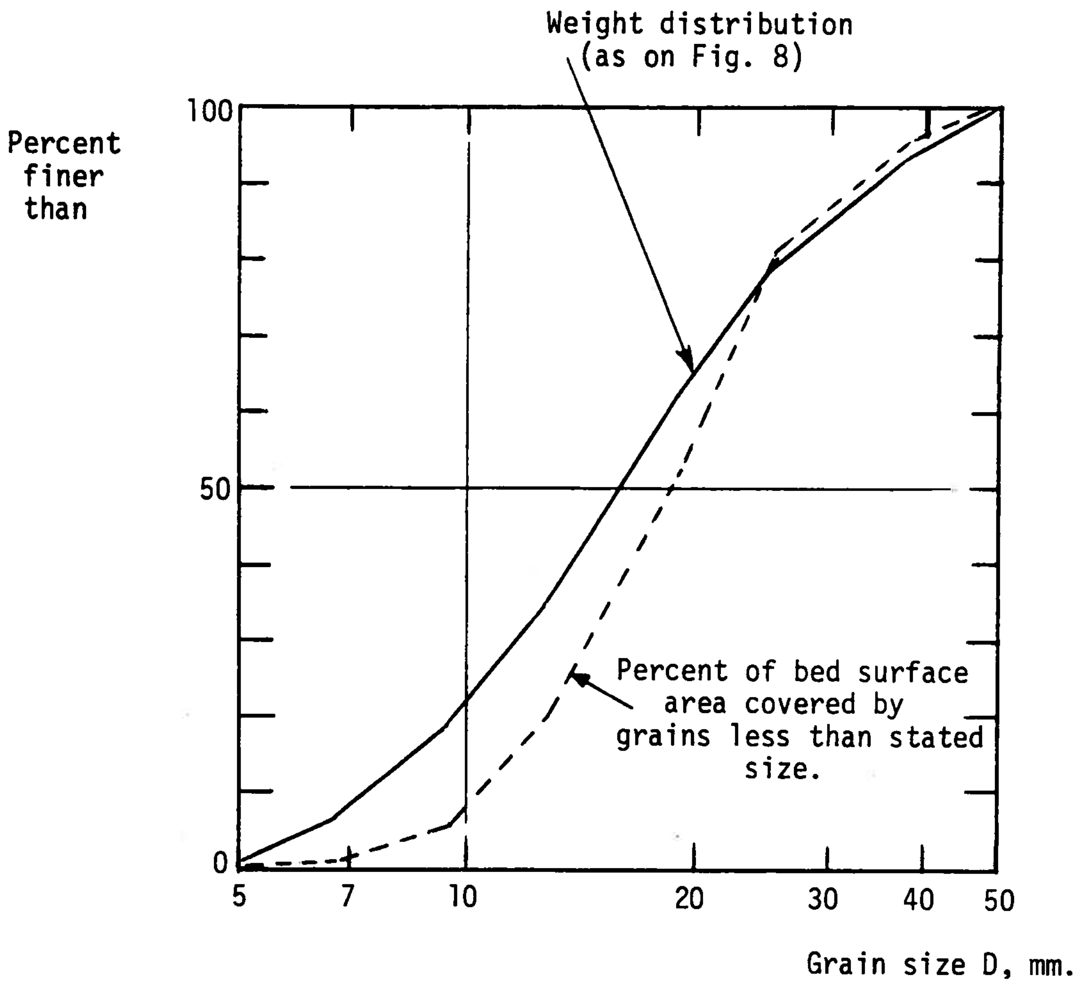
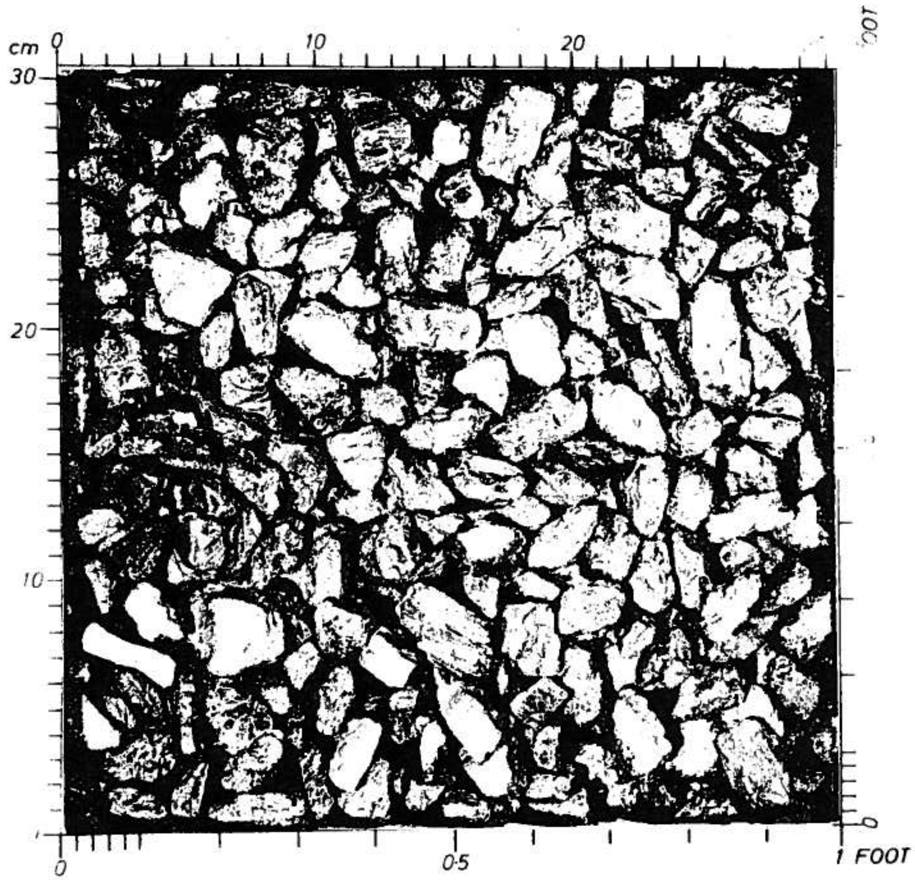
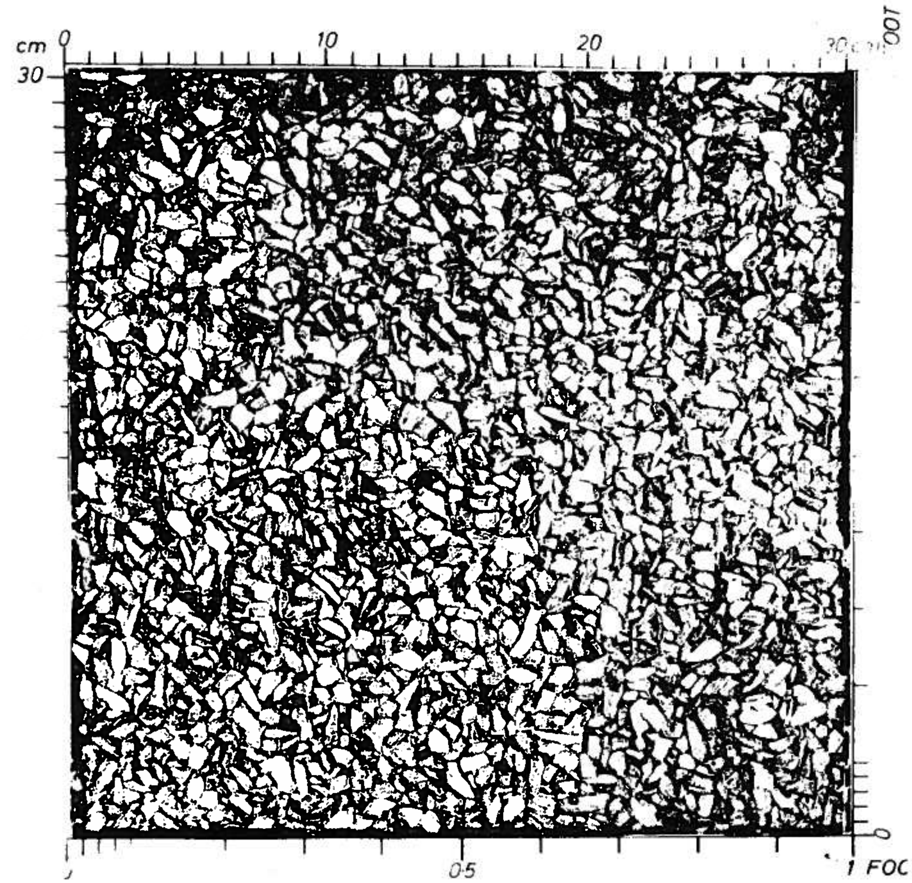


Fig. 9 - Comparison of grain-size distributions by weight (bulk) and by surface area, for mixture M.1.



3/4" -1"

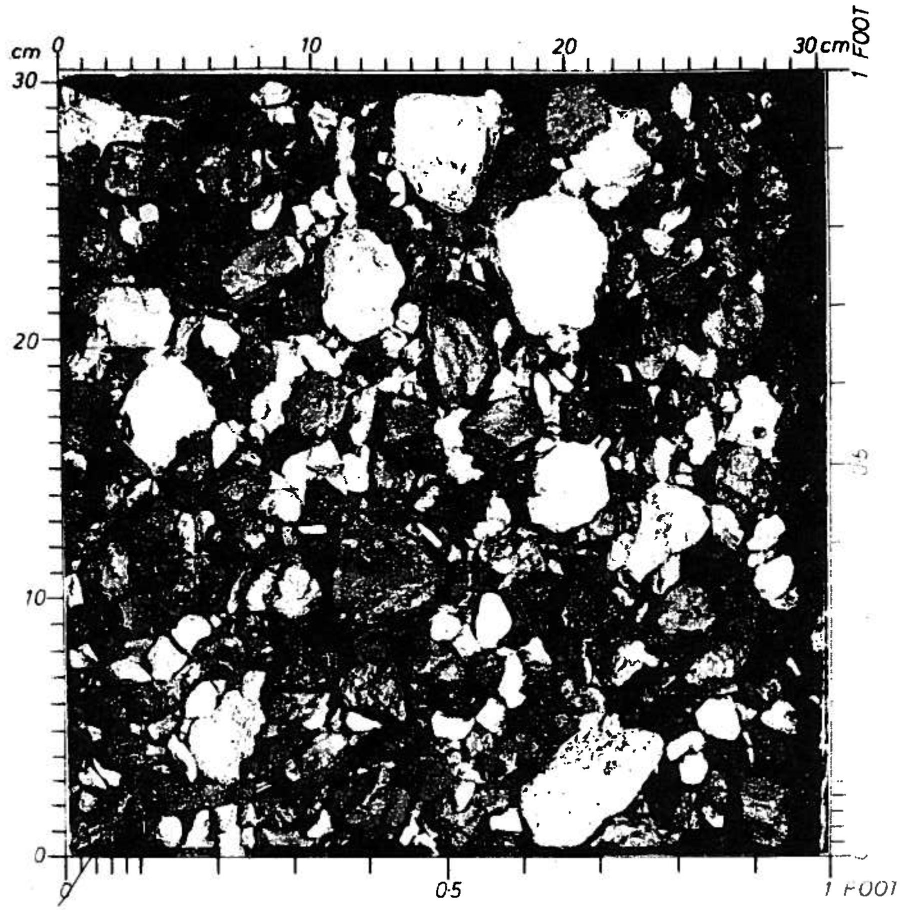
Fraction 'T'



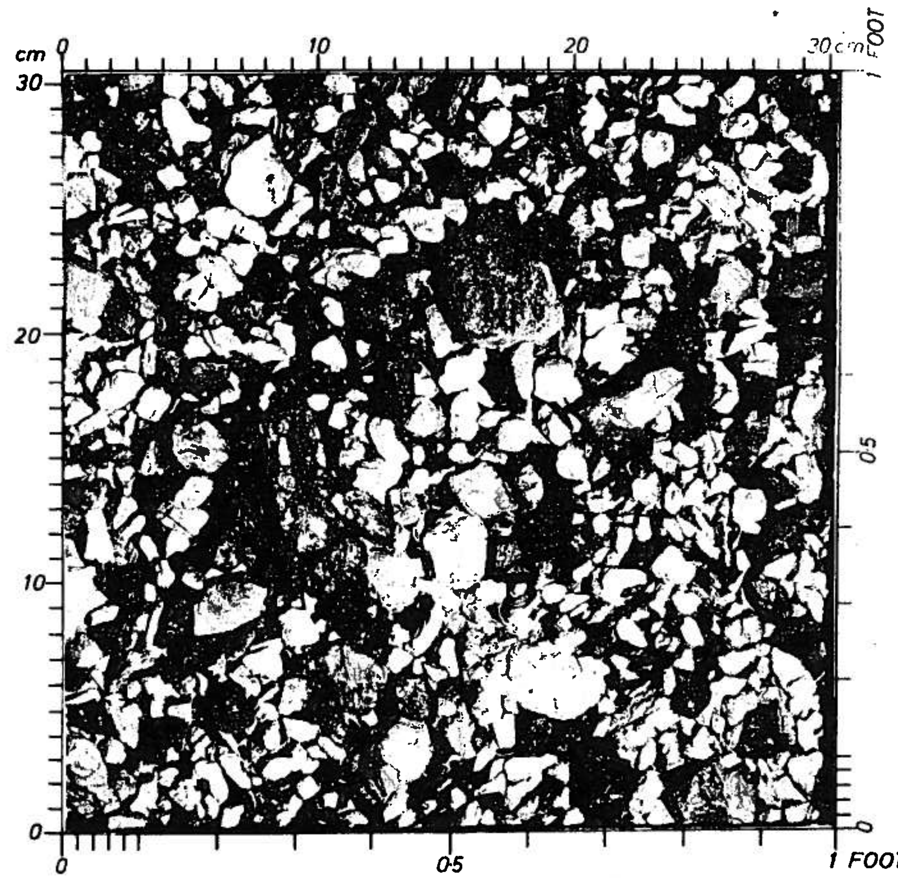
3/16"-1/4"

Fraction 'P'

Fig. 10 - Photo plan views of uniform coal fractions 'T' and 'P'.

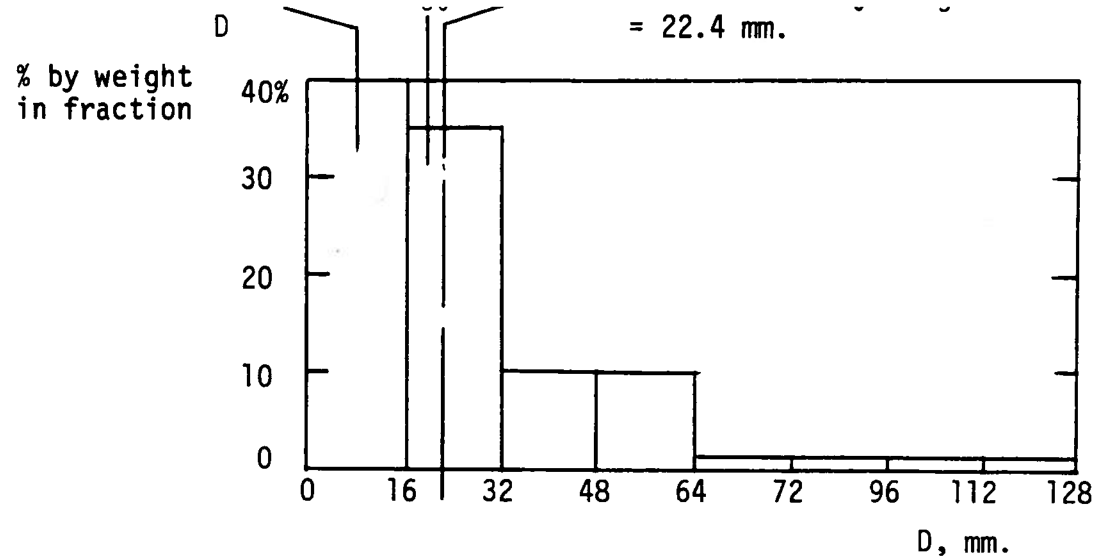


3/16"- 2"
MIXTURE M1

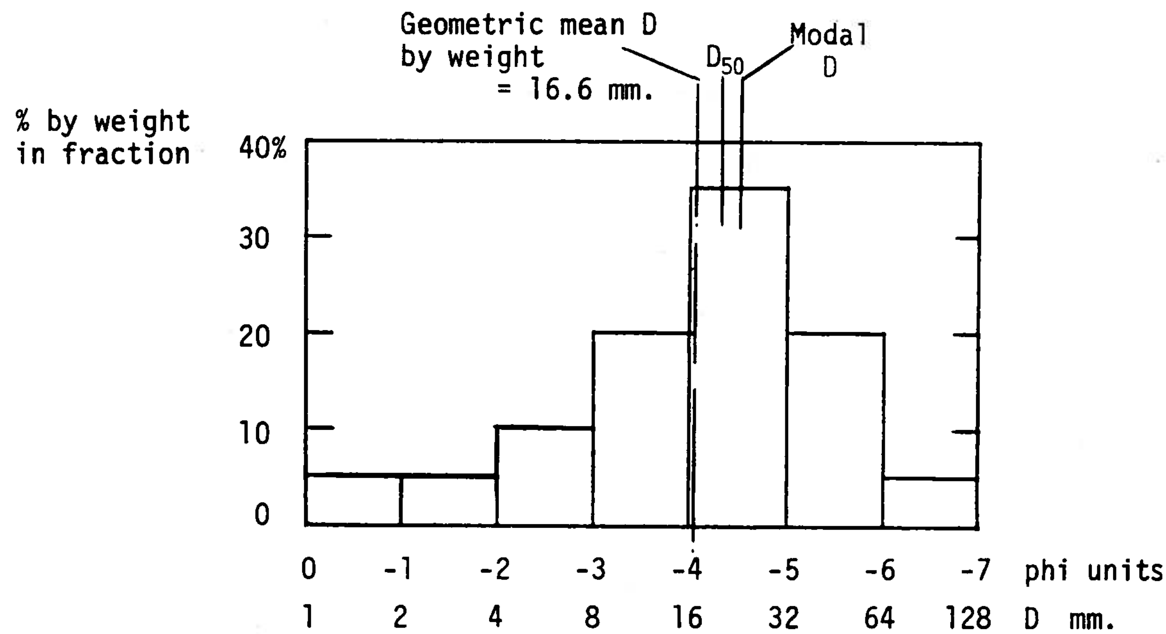


#26 - 2"
MIXTURE M4

Fig. 11 - Photo. plan views of coal mixtures 'M.1' and 'M.4'.

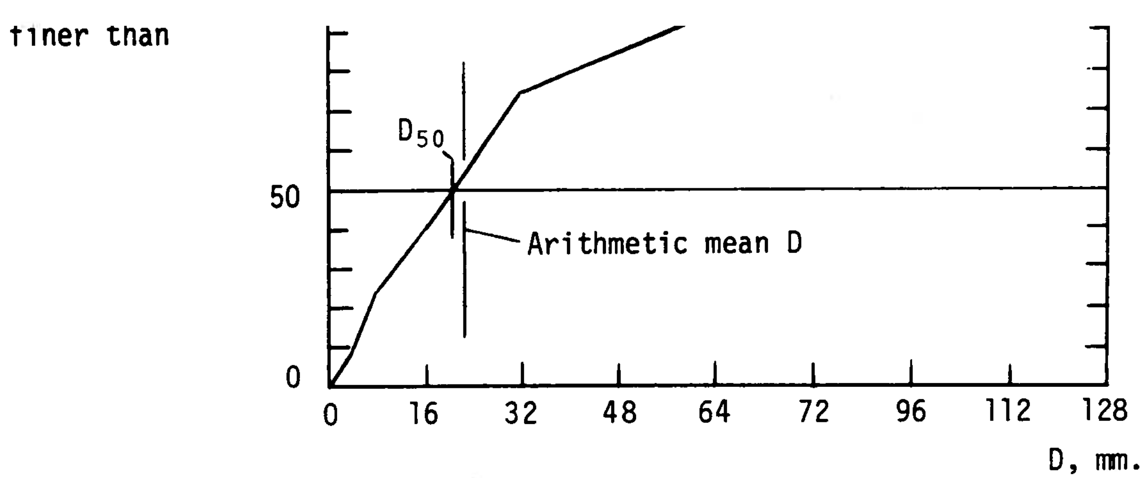


(a) arithmetic scale of grain-size

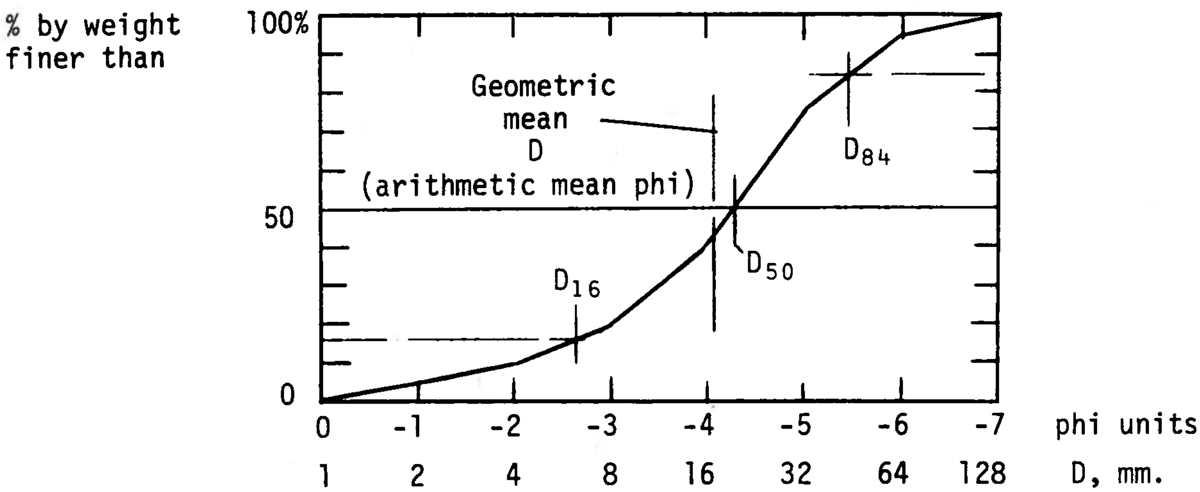


(b) logarithmic scale of grain-size

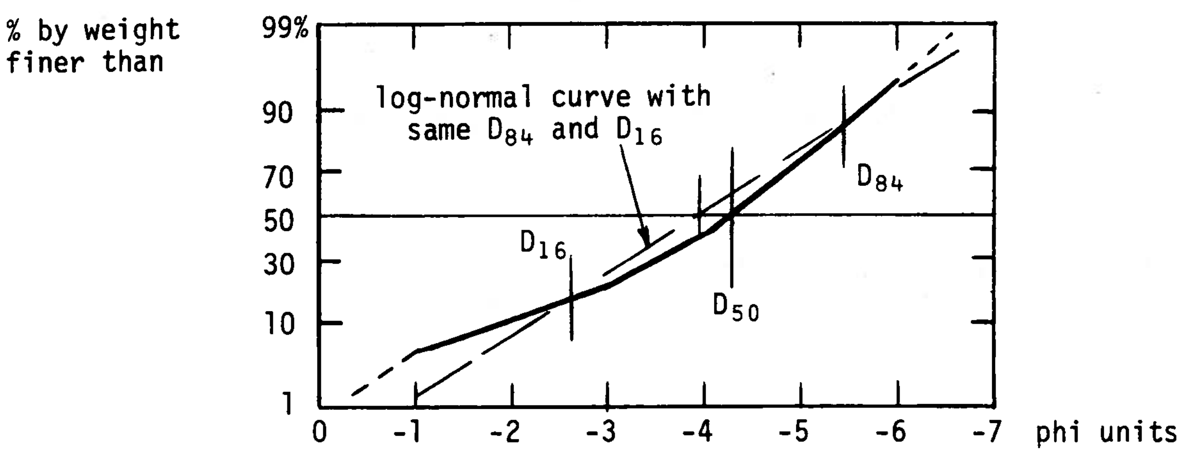
Fig. 12 - Histograms of grain-size distribution for sample of Table 6, showing percentages by weight in various size classes.



(a) arithmetic scale of D



(b) logarithmic scale of D



(c) probability scale of % vs. log. D

Fig. 13 - Cumulative distribution curves for sample of Table 6.

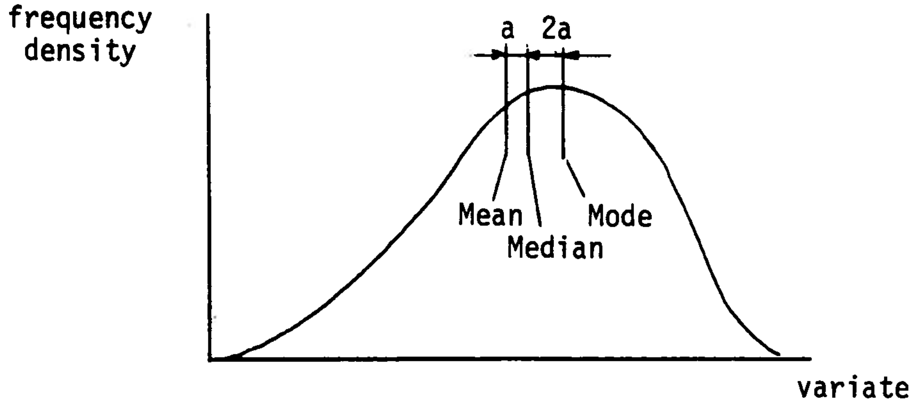


Fig. 14 - Approximate relation between mean, median and mode for moderately skewed probability distribution.

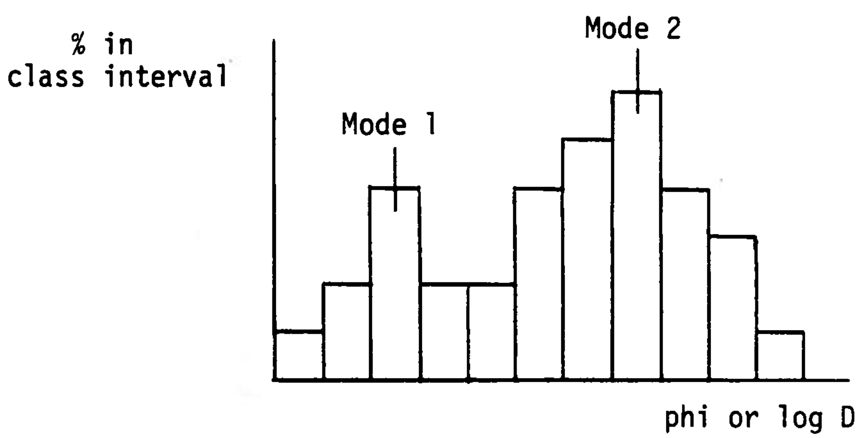


Fig. 15 - Example of bi-modal grain-size distribution, e.g. sand-gravel mixture.

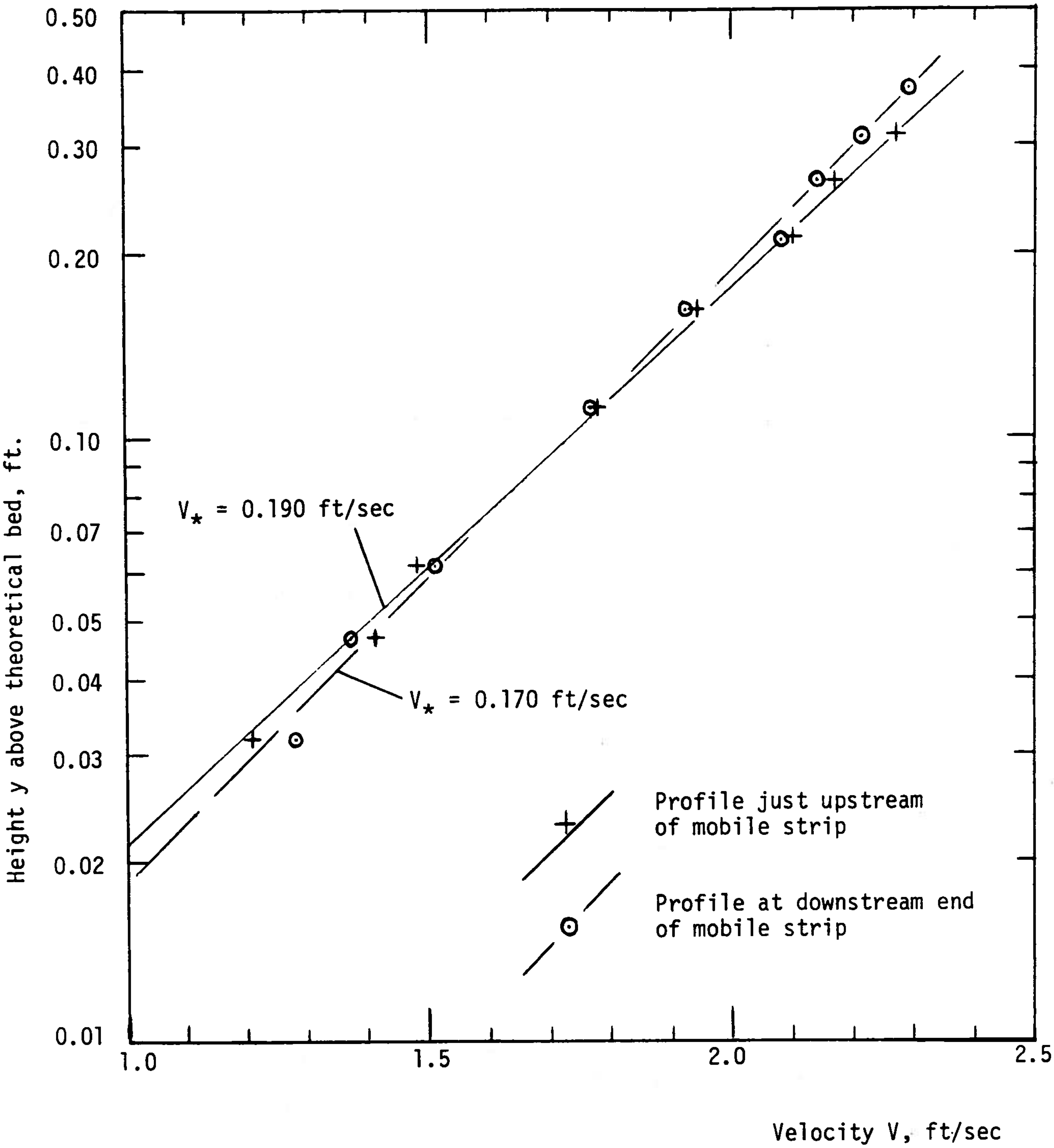


Fig. 16 - Sample midstream velocity profiles from test A.5,

$$S = 0.00212$$

$$\sqrt{ghS} = 0.180 \text{ ft/sec.}$$

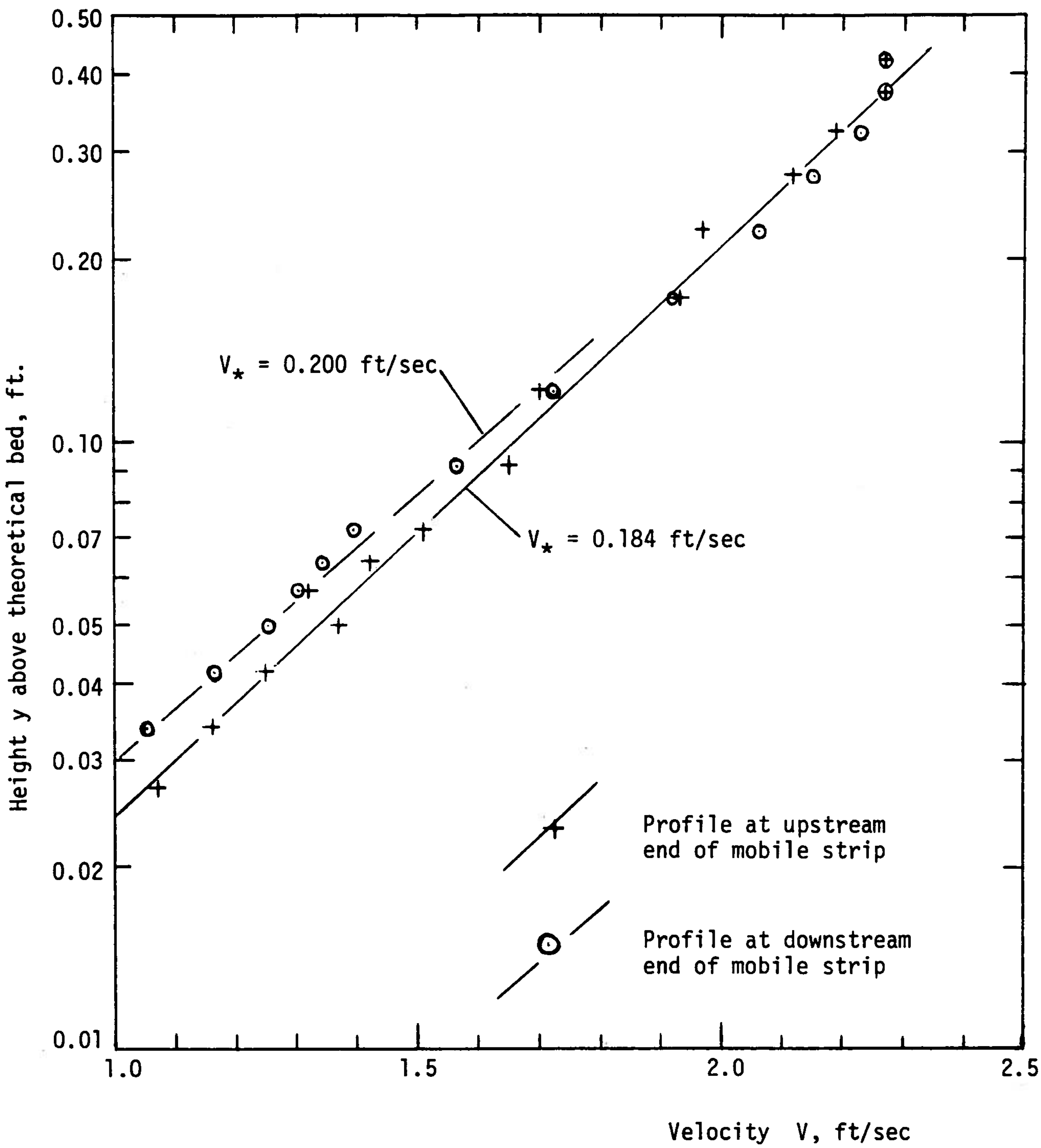


Fig. 17 - Sample midstream velocity profiles from test B.1,

$$S = 0.00200$$

$$\sqrt{ghS} = 0.174 \text{ ft/sec.}$$

TABLE 1 - Summary of material properties and ranges of flow conditions (Wallingford tests)

TEST NO.	MATERIAL PROPERTIES						FLOW CONDITIONS		
	Designation	Nominal Sieve Interval	Dia. or Equivt. Dia.* mm	Submerged Density gm/cc	Average Grain Shape Ratios† c/b	b/a	Depth h mm.	Range of Discharges Q litres/sec	Range of Slopes S per thousand
A.1	Coal fraction 'T'	1" to 3/4"	20.2	0.36	0.64	0.69	145	58	1.51
A.2	"	"	"	"	"	"	61	25	4.00
A.3	"	"	"	"	"	"	170	76	1.47
A.4	"	"	"	"	"	"	144	58 to 89	1.8 to 3.2
A.5	Coal fraction 'S'	1/2" to 3/8"	12.55	0.40	0.62	0.72	144	44 to 72	0.9 to 2.12
A.6	Coal fraction 'P'	1/4" to 3/16"	6.59	0.40	0.55	0.72	144	25 to 57	0.3 to 1.37
A.7	Coal fraction 'N'	#6 to #8	3.45	0.41	0.49	0.67	146	19 to 40	0.18 to 0.65
A.8	"	"	"	"	"	"	144	23 to 28	0.24 to 0.37
B.1	Coal mixture 'M.1"	2" to 3/16"	See grading curve - Fig. 8				144	40 to 82	0.65 to 2.7
B.2	Coal mixture 'M.2'	"	"	"	"		144	52 to 76	1.2 to 2.3
B.3	Coal mixture 'M.3'	2" to #14	"	"	"		144	19 to 71	0.18 to 2.0
B.4	Coal mixture 'M.4'	"	"	"	"		144	25 to 71	0.3 to 2.0
C.1	Glass balls	-	6.0	1.59	1.00	1.00	144	66 to 89	1.8 to 3.2
C.2	"	-	9.0	1.52	"	"	144	82 to 99	2.8 to 4.1

* Equivalent diameter for coal grains taken as diameter of sphere with same average volume. Average volume determined from sample of 100 grains.

† Determined from sample of 25 grains.

TABLE 2 - Summary of principal experimental results

TEST No.	Material	Dia. or equivt. dia. D mm.	Condition of movement	Estimated bed shear stress τ_o kg/m² by flume slope γhS	by velocity profile ρV_*^2	location of profile	Adopted value of τ_o kg/m²	Non-dimensional parameters X = $V_* D/\nu$	Y = $\tau_o/\gamma_s' D$
A.1	Uniform coal	20.2	Lower critical	0.220	0.188	Either end of mobile area	0.205	810	0.028
A.2	"	"	" "	0.244	0.188	Upstream end of mobile area	0.215	830	0.030
A.3	"	"	" "	0.249	0.244	Downstream end of area	0.244	890	0.034
A.4	"	"	" "	0.202	Not measured		0.200	-	0.028
			Upper critical	0.460	0.566	Upstream end	0.488	1250	0.067
					0.488	Downstream end			
A.5	"	12.55	Lower critical	0.180	Not measured at this exact condition		0.180	474	0.036
			Upper critical	0.305	0.342	75 mm Upstream of mobile area	0.308	620	0.061
					0.274	Downstream end			
A.6	"	6.59	Lower critical	0.094	0.075	Upstream end	0.066	151	0.025
					0.058	Downstream end			
			Upper critical	0.197	0.142	75 mm Upstream	0.132	212	0.050
					0.125	Downstream end			
A.7	"	3.45	Lower critical	0.054	0.038	Centre of mobile area	0.038	59	0.027
			Upper critical	0.095	0.087	"	0.087	90	0.062
A.8	"	"	Lower critical	0.054	0.033	Upstream end	0.033	55	0.024
C.1	Glass balls	6.0	Lower critical	0.317	0.322	Centre of mobile area	0.320	320	0.034
			Upper critical	0.389	0.369	Upstream end	0.379	350	0.040
C.2	"	9.0	Lower critical	0.459	0.390	Either end	0.425	550	0.031
				0.591	0.445	Centre	0.518	610	0.038
B.1	Coal mixture	16.0 (D_{50})	Lower critical	0.201	Not measured		0.201	-	0.032
			Upper critical	0.391	0.454	Centre	0.422	-	0.066
B.2	"	16.0 (D_{50})	Lower critical	0.173	Not measured		0.173	-	0.027
			Upper critical	0.332	"	"	0.332	-	0.052
B.3	"	13.0 (D_{50})	Lower critical	0.144	Not measured		0.144	-	0.028
			Upper critical	0.259	0.313	Centre	0.286	-	0.055
B.4	"	8.0 (D_{50})	Lower critical	0.115	0.137	Centre	0.126	-	0.039
			Upper critical	0.259	Not measured		0.259	-	0.081

TABLE 3 - Uniform coal fractions : critical values

Fraction designation	Values of non-dimensional parameters			
	Lower critical stage		Upper critical stage	
	X	Y	X	Y
N	60	0.025	90	0.062
P	150	0.025	210	0.050
R	470	0.036	620	0.061
T	830	0.030	1250	0.067
Mean values		0.029		0.060

TABLE 4 - Uniform glass balls : critical values

Diameter mm.	Values of non-dimensional parameters			
	Lower critical stage		Upper critical stage	
	X	Y	X	Y
6.0	320	0.034	350	0.040
9.0	550	0.031	610	0.038

TABLE 5 - Coal mixtures : critical values

Mixture designation	D_{50} mm	$\sqrt{\frac{D_{84}}{D_{16}}}$	Values of $Y = \frac{\tau_o}{\gamma_s' D_{50}}$	
			Lower critical stage	Upper critical stage
M.1	16	1.83	0.032	0.066
M.2	16	2.27	0.027	0.052
M.3	13	2.76	0.028	0.055
M.4	8	2.48	0.039	0.081
Mean values			0.032	0.064

Hypothetical grain size distribution
with calculated mean values and
dispersion coefficient

Given distribution:

Grain-size D mm.	1	2	4	8	16	32	64	128
Phi units	0	-1	-2	-3	-4	-5	-6	-7
% by weight in size interval	5	5	10	20	35	20	5	
% by weight finer than	0	5	10	20	40	75	95	100

Arithmetic mean by moments of Fig. 12(a)

40	x 0.5	20
35	x 1.5	52.5
10	x 2.5	25
10	x 3.5	35
1.25	x 4.5)	
1.25	x 5.5)	7.5
1.25	x 6.5)	
1.25	x 7.5)	

Mean D = 140 x 16/100 = 22.4 mm

Geometric mean by moments of Fig. 12(b)

5	x 0.5	2.5
5	x 1.5	7.5
10	x 2.5	25
20	x 3.5	70
35	x 4.5	157.5
20	x 5.5	110
5	x 6.5	32.5

Mean phi = 405 /100 = (-)4.05

Corresponding geometric mean D = $2^{4.05}$ = 16.6 mm

Dispersion coefficient from Fig. 13(b)

$$1/2\ (\phi_{84} - \phi_{16}) = 1/2\ (5.45 - 2.60) = 2.85$$

Corresponding $\sqrt{D_{84}/D_{16}} = 2^{2.85} =$ 7.2

TABLE 7 - Test A.5 (uniform coal) : sequence of flow conditions and observed grain movements
(For details of material, mobile area etc. see Table 1 and Fig. 5 : flow depth constant = 0.472 ft.)

Time p.m.	Discharge Q cu.ft/sec.	Slope S per thousand	No. of grains displaced in time interval	Total number of grains displaced from mobile strip	Designated condition of movement
2.45	1.70	1.00		1	
3.05	1.87	1.15			
3.10					'Lower critical' stage interpolated for S = 1.25 per thousand
3.20	2.01	1.40			
3.25			2	3	
3.30			4	7	
3.35			3	10	
3.40	2.17	1.60			
3.45			5	15	
3.50			3	18	
3.55	2.37	1.85			
4.00			5	23	
4.05			6	29	
4.10			6	35	
4.15			1	36	
4.20			2	38	
4.25	2.54	2.12	10	48	Adopted as 'upper critical' stage.
4.30			18	66	
4.35			9	75	
4.40			4	79	
4.45			5	84	

TABLE 8 - Test B.1 (coal mixture) : sequence of flow conditions and observed grain movements

(For details of material, mobile area etc. see Table 1 and Figs. 5 and 8 : flow depth constant = 0.472 ft.)

Time	Discharge Q cu.ft/sec.	Slope S per thousand	No. of grains of various fractions displaced in time interval from mobile strip							Designated condition of movement
			6 mm.	8 mm.	12 mm.	16 mm.	20 mm.	32 mm.	44 mm.	
10.05 a.m.	1.85	1.20								
10.15				(1)	(1)	(2)				(Movement within
10.25					(1)					strip only).
10.40	2.04	1.40								Adopted as
10.45				1	2	1				'lower critical'
10.50							1			stage.
11.05						1				
11.15	2.17	1.60				1				
11.20				1		1	1			
11.30				1	1					
11.35	2.34	1.80			1		1			
11.40					2	2				
11.45			1	1	2	1	2			
11.50						2	1			
11.55						2				
12.00					1	2	1			
12.05 p.m.	2.52	2.00								
12.10					1	7	2			
12.15				1	1	3				
12.25			3	2	4	10	1			
1.25			3	3	12	18	2			
4.25			8	15	8	24	17	1		
4.55	Flow shut down									
10.10 a.m.	Flow resumed									
10.20	2.52	2.00		2	3	2				
10.30				3	1	8	7	1		
10.40			2	3	11	13	5			
10.50			3	4	4	14	4	1		
11.00				2		2				
11.10				3	1	6	1			
11.30				3	1	1		1		
11.50				3	1	5	2			
12.00				3	1	2	1			
12.30 p.m.				2	1	3				
1.30			1	8	4	14	1			
1.45	2.89	2.70								Adopted as
2.15			← Not counted →						1	'higher critical' stage.